greater than steel beads, moved at somewhat greater mean velocities (Fig. 4). It was difficult to establish quantitative agreement between the theory and the experiment, since we did not determine the angle θ for a specific experimental scheme (see the scheme in Fig. 4). However, if we compare Eq. (14) and the test data for the steel beads, then for a certain mean angle we obtain the value $\theta \sim 0.4$ (23°); this corresponds fully to the actual situation.

In conclusion, we should note that the results obtained depend little on the form of the particle. Points 5 in Fig. 4 correspond to steel cubes with an equivalent diameter $d \simeq 3.5$ mm. The moderate difference in the proportionally factor from the case of steel beads (spheres) is possibly connected with the somewhat different values of the resistance and restitution coefficients for the cubes. The geometry of the chamber is evidently the determining factor for the characteristics of particle motion, other conditions being equal as determined by the above-developed theory.

NOTATION

R, h, radius and height of turbulence chamber; θ, angle of rotation of plane of rib relative to a tangent to an inscribed cylinder; **V**, ρ, velocity of gas and its density; d, ρ_1, **w**, ω, diameter of particle, its density, linear velocity, and angular velocity relative to the center of mass; **u**, relative velocity of particle and gas; $z = x + iy$, complex coordinate of particle; t, τ, time; T, period of time between impacts; n, restitution coefficient; ζ, coefficient of resistance of particle; k_1, numerical coefficient in the Magnus force; α, β, dimensionless criteria.

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CONVECTIVE HEAT EXCHANGE IN TURBULENT FLOW OF A GAS SUSPENSION WITHIN A CYLINDRICAL CHANNEL

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A convective heat-exchange model is presented for flow of a gaseous suspension in a tube, which considers the increase in heat capacity of the system and the effect of particles on the turbulent structure of the flow. Comparison of calculated results with experiment shows good agreement.

The intensity and efficiency of many heat-exchange processes in metallurgy, energy generation, and other branches of industry are determined by phenomena occurring in gas—solid particle type dispersed systems. Heat exchange was treated in [1] with consideration of the effect of particles on the turbulent structure of the carrier flow within the framework of Buevich's model [2], which consists of breakoff of the shortwave portion of the turbulent energy spectrum. The calculation results of [1] agree well with experiment in the low particle concentration range $\mu \leqslant 6$. It is of interest to consider the case of higher particle concentrations.

We will write the energy equation for the turbulent flow of a gas suspension in a tube just as in [1], but without consideration of radiation

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$$(1-\beta)c\rho u\frac{\partial T}{\partial x}+\beta c_1\rho_1 u_1\frac{\partial T_1}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left\{(1-\beta)r(\lambda+\lambda_t)\frac{\partial T}{\partial r}-r\beta c_1\rho_1\langle v_1'T_1'\rangle\right\}, \qquad (1)$$

where $\langle v_1'T_1'\rangle$ is a quantity characterizing turbulent energy transfer by particles.

The last term of Eq. (1) can be interpreted as the turbulent heat flux transferred by particles $\beta c_1\rho_1\langle v_1'T_1'\rangle = \beta c_1\rho_1 a_1^*(\partial T_1/\partial r)$, where a_1^* is the turbulent heat diffusion coefficient, related to particle displacement.

To describe heat exchange in the two-phase flow Eq. (1) must be complemented by an energy equation for the particles. In the present study we will assume for simplicity that the temperatures of gas and particles coincide (which is valid for particles small in size). Then in in dimensionless form energy equation (1) takes on the form

$$\frac{u}{u_m}(1+z)\frac{\partial\theta}{\partial\bar{x}}=\frac{4}{\mathrm{Re}}\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left\{\bar{r}\left[\frac{1}{\mathrm{Pr}}+\frac{1}{\mathrm{Pr_t}}\frac{\nu_t}{\nu}+\frac{z}{\varphi}\frac{1}{\mathrm{Pr_t^*}}\frac{\nu_p}{\nu}\right]\frac{\partial\theta}{\partial\bar{r}}\right\}. \qquad (2)$$

Here we introduce the notation $\mathrm{Pr_t}=\nu_t/a_t$; $\mathrm{Pr_t^*}=\nu_p/a_t^*$; $\mathrm{Re}=u_m D/\nu$; $z=\frac{\beta}{1-\beta}\frac{\rho_1}{\rho}\frac{u_1}{u}\frac{c_1}{c}$; $\varphi=u_1/u$; $\bar{x}=x/D$; $\bar{r}=r/R$; $\theta=T/T_0$. The boundary conditions for Eq. (2) are as follows:

$$\begin{aligned}&\bar{x}=0 \quad \theta(0,\bar{r})=1;\\ &\bar{r}=1 \quad \theta(\bar{x},1)=\theta_w;\\ &\bar{r}=0 \quad \frac{\partial\theta}{\partial\bar{r}}=0.\end{aligned} \qquad (3)$$

According to [3], the velocity profile of the carrier medium can be written in the form

$$u/v^*=\frac{2{,}3}{\varkappa_0}\lg y^+ + 5.8; \quad 30\leqslant y^+\leqslant 350;$$
$$u/v^*=y^+; \quad y^+<30,$$

where y is the distance from the wall; $y^+=v^*y/\nu$.

To describe the velocity in the flow core we take the profile of [1]

$$\frac{u_0-u}{v^*}=-\frac{1}{\varkappa_0\sqrt{1+\eta\mu}}\left[\ln\left(1-\sqrt{1-\frac{y}{R}}\right)-\sqrt{1-\frac{y}{R}}\right]; \quad y^+>350,$$

where u_0 is the velocity on the tube axis, and η is a coefficient defined by data from [3].

As before, for the turbulent viscosity profile of the carrier medium we take the model of [2]. For the viscous and transition layers we have

$$\frac{\nu_t}{\nu}=\varkappa_0^2\frac{\mathrm{Re}}{2}(1-\bar{r})^2\left(1-\exp\frac{\bar{r}-\bar{r}_0}{\alpha_1}\right)^2\left|\frac{\partial\bar{u}}{\partial\bar{r}}\right|,$$

where $\bar{r}=r/R$; $\bar{u}=u/u_m$, $\alpha_1=\alpha\nu/Rv^*$. From the data of [2] $\alpha=30.4$, $\bar{r}_0$ is calculated with the expression

$$\bar{r}_0=1-a\mu d_1/D, \qquad (4)$$

where a is a proportionality coefficient.

We write the turbulent diffusion coefficient for the solid particles in the form of a ratio [4]

$$\nu_p=\left(\frac{\rho}{\rho_1}\right)^{2/3}\nu_t, \qquad (5)$$

which is valid for cases in which the particle relaxation time is greater than the external time scale of the turbulence.

The problem of Eq. (2) with boundary conditions (3) was solved numerically on a BÉSM-6 computer. Calculations of dynamic gas velocity v* were performed by the method described in [3]: as a function of $\mathrm{Re_s}$, based on the rotational velocity and size of the particles, the

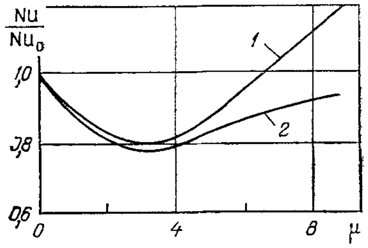

Fig. 1. Dependence of heat liberation on particle concentration: 1) experiment [5]; 2) calculation.

quantity η was determined, after which the shear stress in the carrying medium was found from the expression $\tau_t = (1 + \eta\mu)\tau_0$, where τ_0 is determined from the Blasius law.

Figure 1 shows the results of a calculation for Re = 32,000, D = 18.8 mm, d_1 = 100 μm. The quantity Nu_0 characterizes heat liberation from a pure air flow. Also shown is an experimental curve from [5]. The results agree well for $\mu \leqslant 6$, but differ strongly at higher concentrations. This disagreement is apparently due to the fact that aside from suppression of high-frequency pulsations of the carrier fluid by particles, with increase in concentration there must be some "generation" of turbulence due to formation of wakes as particles are flowed over. It was noted in [6] that due to difference in the velocities of fluid and particles a system of vortices (shroud) is formed, the presence of fine scale vortices in which changes the form of the turbulent energy spectrum in the range of wave numbers comparable to $1/d_1$. In order to introduce an additional turbulent viscosity for the carrier medium, generated by the system of wakes, we may use the well-known relationships of Kolmogorov [7] and Prandtl [8];

$$\varepsilon = \text{const}\, e^{3/2}/l_{\text{к}}, \tag{6}$$

$$\nu' = \text{const}\, e^{1/2} l_{\text{к}}. \tag{7}$$

It follows from Eqs. (6), (7) that

$$\nu' = \text{const}\, \varepsilon^{1/3} l_{\text{к}}, \tag{8}$$

i.e., the additional turbulent viscosity is expressed in terms of the energy dissipation to particles ε, while for the scale length l_k we may take a quantity proportional to the particle diameter.

We will assume approximately that "generation" of turbulent energy in particle wakes is equal to the accuracy of a coefficient to the viscosity dissipation. Then the mean energy dissipation per unit mass of fluid can be expressed by

$$\varepsilon' = \text{const}\, \nu\rho d_1 \langle (u' - v')^2 \rangle, \tag{9}$$

where u', v' are velocity pulsations of fluid and particles, respectively. We write the difference in pulsation velocities in the form of [9]

$$\langle u_R^2 \rangle = \langle (u' - v')^2 \rangle = \langle u'^2 \rangle \int_0^\infty \frac{\Omega_R}{\Omega_2} f(\omega)\, d\omega; \tag{10}$$

where

$$\Omega_R = \frac{1-b}{b} \frac{\omega}{\alpha};$$

$$\Omega_2 = \frac{1}{b^2}\left(\frac{\omega}{\alpha}\right)^2 + \frac{\sqrt{6}}{b}\left(\frac{\omega}{\alpha}\right)^{3/2} + 3\left(\frac{\omega}{\alpha}\right) + \sqrt{6}\left(\frac{\omega}{\alpha}\right)^{1/2} + 1;$$

$$b = \frac{3\rho}{2\rho_1 + \rho}; \quad \alpha = \frac{12\nu_0}{d_1^2};$$

$f(\omega)$ is a Lagrangian function of the energy spectrum.

The expression for the total dissipation on all particles will then be

$$\varepsilon = \text{const}\, \mu \frac{\rho}{\rho_1} \frac{\nu}{d_1^2} \langle u'^2 \rangle \int_0^\infty \frac{\Omega_R}{\Omega_2} f(\omega)\, d\omega. \tag{11}$$

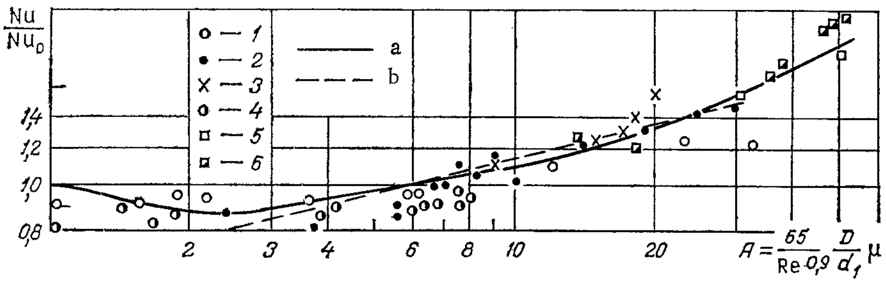

Fig. 2. Comparison of calculated results for heat liberation in the stabilized region with various experiments: 1) [11]; 2) [12]; 3) [13]; 4) [14]; 5) [15]; 6) [16]; a) calculation; b) generalization of [5].

Substituting this expression in Eq. (8) we obtain

$$\frac{\nu'_t}{\nu} = \text{const}\,\mu^{1/3}\left(\frac{\rho}{\rho_1}\right)^{1/3}\left(\frac{\langle u'\rangle d_1}{\nu}\right)^{2/3}\left[\int_0^\infty \frac{\Omega_R}{\Omega_2} f(\omega)\,d\omega\right]^{1/3}. \tag{12}$$

We will define <u'> with the analytic expression of [10], which approximates Laufer's experimental data well:

$$\langle u'\rangle = Av^*(y^+)^{3/2}\exp(-0.05y^+), \quad y^+ \leqslant 21.52, \tag{13}$$

$$\langle u'\rangle = \frac{y^+}{\left(\frac{0.53}{R_+}\right)y^{+^2} + 0.85y^+ + 14}, \quad y^+ > 21.52. \tag{14}$$

Expressions for calculation of A and R_+ are presented in [10].

Thus, the additional turbulent viscosity ν'_t appears in Eq. (2) as a term in ν_t. In calculating the value of the integral in Eq. (12) data from [9] were used for the energy spectrum. The results of solving Eq. (2) with consideration of additional viscosity (12) were processed in the form of the dependence of Nu/Nu_0 on the complex

$$A = \frac{65}{Re^{0.9}}\frac{D}{d_1}\mu.$$

Figure 2 shows the calculated dependence of Nu/Nu_0 on μ. The points indicate experimental data of various researchers. These calculated results were obtained at $Pr_t^* = 1$; $a = 2.9$ and with the constant in Eq. (12) equal to 8.348. The dashed line is the empirical expression of [5]. The convective heat-exchange model considered herein is simple and gives good agreement with the experiments of [11-16].

NOTATION

β, volume concentration of solid phase; λ, λ_t, molecular and turbulent thermal conductivity of carrier medium; c, ρ, c_1, ρ_1, specific heat and density of gas and particles; ν, kinematic viscosity; Pr_t, turbulent Prandtl number; D, channel diameter; T_0, gas temperature at entrance; u_m, mean flow velocity; $Re = u_m D/\nu$, Reynolds number; v^*, dynamic viscosity; $\varkappa_0$, Karman constant; μ, solid phase concentration by weight (kg/kg); d_1, particle diameter; e, turbulent energy; ε, energy dissipation; l_k, turbulence scale; ω, frequency.

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WAVE STRUCTURE OF TURBULENT FLOW IN A TUBE

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The wave theory of turbulence is applied to determine the fluctuation field u of shear flow in a tube.

The particle-wave representation of turbulence reflects several of its quantum-mechanical properties: the fluctuation fields of vortices are manifested as a fluctuation probability wave, encompassing the region of the statistically coupled vortex state. The wavelength of the probability standing wave determines the largest vortex size occurring in the transverse flow scale. A mean (regular) shear flow is realized within the limits of this wave. In the shear model the fluctuation field is represented by means of the wave function ψ, determining the probability wave amplitude — the fluctuation intensity, as well as their linear scales inversely proportional to the wave number. The system of equations and the foundations of the method discussed were published earlier in [1, 2]. Several assumptions on the quantum analogies of turbulence were discussed in [3].

Turbulent flow in a tube at a sufficient distance from its input cross section is realized without longitudinal variation of the fluctuation field. In this case

$$ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2\rho}\left(\frac{\partial^2\psi}{\partial y^2} + \frac{1}{y}\frac{\partial\psi}{\partial y}\right), \quad i = \sqrt{-1}. \tag{1}$$

The behavior of the ψ-wave, described by Eq. (1), is different near the wall ($y \to R$) and near the flow axis ($y \to 0$), since the vortex structure is inhomogeneous in these two regions. The increase in fluctuation intensity at the wall, along with enhanced tendency toward vortex formation, implies existence of an inhomogeneous wave at the walls. The stabilized structure of vortices "torn" from the wall is characteristic of the central flow region, in which the fluctuation level is also stable. The inhomogeneous wave corresponds to the special representation $\psi = a\exp(ib)$, so that for stationary turbulence ($\partial a^2/\partial t = 0$) we obtain, according

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